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Special uniform approximations of continuous vector-valued functions. Part II: special approximations in $C_X(T) \otimes C_Y(S)$

Vlad Timofte*

Département de Mathématiques, École Polytechnique, Fédérale de Lausanne, 1015 Lausanne, Switzerland

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Abstract

In this paper, which is a continuation of Timofte (J. Approx. Theory 119 (2002) 291–299, we give special uniform approximations of functions from $C_{X\otimes Y}(T \times S)$ and $C_\infty(T \times S, X \otimes Y)$ by elements of the tensor products $C_X(T) \otimes C_Y(S)$, respectively $C_0(T, X) \otimes C_0(S, Y)$, for topological spaces T, S and Γ-locally convex spaces X, Y (all four being Hausdorff).

Keywords: Uniform approximation; Tensor product

1. Introduction

We will use the symbol Γ to denote one of the fields $\mathbf{R}, \mathbf{C}$. It is known that if T, S are non-empty sets and X, Y are Γ-vector spaces, then the application $\theta : F_X(T) \otimes F_Y(S) \to F_{X\otimes Y}(T \times S)$,

$$\theta\left(\sum_{i\in I} f_i \otimes g_i\right)(t,s) = \sum_{i\in I} f_i(t) \otimes g_i(s)$$

is well defined, linear and injective (where $F_X(T)$ denotes the vector space of all X-valued functions on T).

From now on, we consider two topological spaces T, S, two Γ-locally convex spaces X, Y (all four Hausdorff) and a Hausdorff locally convex topology on

*Fax: 0041-21-693-5839.

E-mail address: vlad.timofte@epfl.ch.

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$Z := X \otimes Y$, such that the bilinear application $\otimes : X \times Y \to Z$ is continuous. Let $M := T \times S$ denote the product topological space. Since $\theta(C_X(T) \otimes C_Y(S)) \subset C_Z(M)$, we have by some natural identifications the following inclusions:

$$(C_\Gamma(T) \otimes X) \otimes (C_\Gamma(S) \otimes Y) \subset C_X(T) \otimes C_Y(S) \subset C_Z(M),$$

$$(C_0(T,\Gamma) \otimes X) \otimes (C_0(S,\Gamma) \otimes Y) \subset C_0(T,X) \otimes C_0(S,Y) \subset C_\infty(M,Z).$$

For abbreviation, we will use the following notations:

$$E := (C_\Gamma(T) \otimes X) \otimes (C_\Gamma(S) \otimes Y), \qquad E_0 := (C_0(T,\Gamma) \otimes X) \otimes (C_0(S,\Gamma) \otimes Y).$$

By the natural algebraical isomorphisms

$$E \simeq (C_\Gamma(T) \otimes C_\Gamma(S)) \otimes Z, \quad E_0 \simeq (C_0(T,\Gamma) \otimes C_0(S,\Gamma)) \otimes Z,$$

we will identify the corresponding vector spaces. Various results concerning the uniform density of $C_\Gamma(T) \otimes X$ in $C_X(T)$, of $C_X(T) \otimes C_Y(S)$ in $C_Z(M)$ and Weierstrass–Stone's type theorems are known (see [1–6]). Therefore, we will restrict our attention to special uniform approximations in $C_Z(M)$ and in $C_\infty(M,Z)$. This concept was introduced in [7].

Definition 1. Let $u \in C_Z(M)$ (respectively, $u \in C_\infty(M,Z)$) and the neighborhood $W \in \mathscr{V}_Z(0)$ be fixed. A function

$$u_W \in C_X(T) \otimes C_Y(S) (\text{respectively, } u_W \in C_0(T,X) \otimes C_0(S,Y))$$

is said to be a special W-uniform approximant of u, if and only if u_W satisfies $(u - u_W)(M) \subset W$ and the following constraints:

$$u_W(M) \subset \mathrm{co}(u(M)) (\text{respectively, } u_W(M) \subset \mathrm{co}(u(M) \cup \{0\})),$$

$$\mathrm{supp}\, u_W \subset u^{-1}(Z \backslash \{0\}).$$

If $X = \Gamma$ and $S = \{s\}$ is a singleton, then M is homeomorphic to T. Since $Z \simeq Y$ as locally convex spaces, using the algebraical isomorphisms

$$C_Z(M) \simeq C_Y(T), \quad C_X(T) \otimes C_Y(S) \simeq C_\Gamma(T) \otimes Y,$$

$$C_\infty(M,Z) \simeq C_\infty(T,Y), \quad C_0(T,X) \otimes C_0(S,Y) \simeq C_0(T,\Gamma) \otimes Y,$$

reduces the above definition to the notion studied in [7]. To make our exposition self-contained, we repeat a needed result (see [7, Theorem 1, p. 293]) without proof, in a particular setting:

Theorem 1. *If T is compact, then for all $u \in C_X(T)$ and $W \in \mathscr{V}_X(0)$, there exists an approximant $u_W \in C_\Gamma(T) \otimes X$, such that*:

(1) $(u - u_W)(T) \subset W,\ u_W(T) \subset \mathrm{co}(u(T)), \mathrm{supp}\, u_W \subset u^{-1}(X \backslash \{0\})$,
(2) $u_W = \sum_{i \in I} \varphi_i(\cdot) x_i$ *for some finite set I, $(x_i)_{i \in I} \subset u(T)$ and $(\varphi_i)_{i \in I}$ p.u.* (*partition of unity*) on T.

2. Special approximations in $C_Z(M)$

2.1. The compact case of $E \subset C_X(T) \otimes C_Y(S) \subset C_Z(M)$

Only in this subsection we assume T and S to be compact spaces.

Lemma 1. *If $K \subset D \subset M$, K is closed and D is open, then for every $\varepsilon \in (0,1)$, there exists $\omega \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$, such that*

$$0 \leqslant \omega < 1, \quad \omega(\xi) > 1 - \varepsilon \ \ \forall \xi \in K, \quad \operatorname{supp} \omega \subset D.$$

Proof. Obviously, $\exists (U_i)_{i\in I}$, $\exists (V_i)_{i\in I}$ finite families of open subsets of T, respectively S, such that $K \subset \bigcup_{i\in I} (U_i \times V_i) \subset \bigcup_{i\in I} (\overline{U_i} \times \overline{V_i}) \subset D$. Since $M = (M \backslash K) \cup \bigcup_{i\in I} (U_i \times V_i)$, using a p.u. subordinated to the previous open covering gives that $\exists (\varphi_i)_{i\in I} \subset C_{\mathbf{R}}(M)_+$, such that $\sum_{i\in I} \varphi_i \leqslant 1$, $\sum_{i\in I} \varphi_{i_{|_K}} \equiv 1$ and $\operatorname{supp} \varphi_i \subset U_i \times V_i$ $\forall i \in I$. Set $\delta := \frac{\varepsilon}{3}, \rho := \frac{\varepsilon}{3|I|} = \frac{\delta}{|I|}$ and fix $i \in I$. By Stone–Weierstrass' theorem, $\exists \psi_i \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$, such that $\|(1-\delta)\varphi_i + \rho/2 - \psi_i\|_\infty < \rho/2$. Therefore, on M we have the pointwise inequalities $0 \leqslant (1-\delta)\varphi_i < \psi_i < (1-\delta)\varphi_i + \rho$. Now consider the compact set $K_i := \{\xi \in M \mid \varphi_i(\xi) \geqslant \rho\} \subset \operatorname{supp} \varphi_i \subset U_i \times V_i$. By Urysohn's lemma, $\exists a_i \in C_{\mathbf{R}}(T)$, $b_i \in C_{\mathbf{R}}(S)$, such that $0 \leqslant a_i \leqslant 1$, $0 \leqslant b_i \leqslant 1, \operatorname{supp} a_i \subset U_i, \operatorname{supp} b_i \subset V_i$, $(a_i \otimes b_i)_{|_{K_i}} \equiv 1$. Define $\omega_i := (a_i \otimes b_i)\psi_i \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$, $\omega := \sum_{i\in I} \omega_i \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$. We obviously have $\operatorname{supp} \omega \subset \bigcup_{i\in I}$ *supp* $\omega_i \subset \bigcup_{i\in I} \operatorname{supp}(a_i \otimes b_i) \subset \bigcup_{i\in I} (U_i \times V_i) \subset D$ and $0 \leqslant \omega = \sum_{i\in I} \omega_i \leqslant \sum_{i\in I} \psi_i < (1-\delta)\sum_{i\in I} \varphi_i + \rho|I| \leqslant 1$. The proof is completed by showing that $\omega_{|_K} > 1 - \varepsilon$. Fix $\xi \in K$ and consider $I_\xi := \{i \in I \mid \xi \in K_i\} = \{i \in I \mid \varphi_i(\xi) \geqslant \rho\}$. It follows that $\omega(\xi) \geqslant \sum_{i\in I_\xi} \omega_i(\xi) = \sum_{i\in I_\xi} \psi_i(\xi) = \sum_{i\in I} \psi_i(\xi) - \sum_{i\in I\backslash I_\xi} \psi_i(\xi) > (1-\delta)\sum_{i\in I} \varphi_i(\xi) - \sum_{i\in I\backslash I_\xi}[(1-\delta)\varphi_i(\xi) + \rho] \geqslant 1 - \delta - [(1-\delta)\rho + \rho] \cdot |I| = 1 - \delta - \delta(2-\delta) > 1 - 3\delta = 1 - \varepsilon$. Hence, ω satisfies all required properties. □

Theorem 2. *If $u \in C_Z(M)$, then for every $W \in \mathscr{V}_Z(0)$, there exists an approximant $u_W \in E$, such that*

$$(u - u_W)(M) \subset W, \quad u_W(M) \subset \operatorname{co}(u(M)), \operatorname{supp} u_W \subset u^{-1}(Z \backslash \{0\}).$$

Proof. Fix $W \in \mathscr{V}_Z(0)$ and choose $W_0 \in \mathscr{V}_Z(0)$, with W_0 balanced, convex and $2W_0 \subset W$. By Theorem 1, $\exists v = \sum_{i\in I} \varphi_i(\cdot) z_i \in C_\Gamma(M) \otimes Z$, such that $(u - v)(M) \subset W_0$, $v(M) \subset \operatorname{co}(u(M)), \operatorname{supp} v \subset u^{-1}(Z \backslash \{0\})$, $(z_i)_{i\in I} \subset u(M)$, $(\varphi_i)_{i\in I}$ p.u. on M for some finite set I. Since the set $A := \operatorname{co}(u(M))$ is bounded in Z, $\exists \varepsilon \in (0,1)$, with $2\varepsilon A \subset W_0$. For every fixed $i \in I$, by Stone–Weierstrass' theorem, $\exists \psi_i \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$, such that $\|(1-\varepsilon)\varphi_i + \frac{\varepsilon}{2|I|} - \psi_i\|_\infty < \frac{\varepsilon}{2|I|}$. Therefore, on M we have the pointwise inequalities $0 \leqslant (1-\varepsilon)\varphi_i < \psi_i < (1-\varepsilon)\varphi_i + \varepsilon/|I|$, and so $1 - \varepsilon < \sum_{i\in I} \psi_i < 1$. Fix $z \in u(M)$ and define $w_z := \sum_{i\in I} \psi_i(\cdot)(z_i - z) + z = \sum_{i\in I} \psi_i(\cdot) z_i + (1 - \sum_{i\in I} \psi(\cdot))z$. Thus, $w_z \in E$, $w_z(M) \subset \operatorname{co}(u(M))$. We next show that $(w_z - v)(M) \subset W_0$. For every

$\xi \in M$, we have $(w_z - v)(\xi) = \sum_{i \in I}(\psi_i(\xi) - \varphi_i(\xi))z_i + (1 - \sum_{i \in I}\psi_i(\xi))z = \sum_{i \in I}[\psi_i(\xi) - (1-\varepsilon)\varphi_i(\xi)]z_i - \varepsilon \sum_{i \in I}\varphi_i(\xi)z_i + (1 - \sum_{i \in I}\psi_i(\xi))z \in \sum_{i \in I}[\psi_i(\xi) - (1-\varepsilon)\varphi_i(\xi)]A - \varepsilon A + (1 - \sum_{i \in I}\psi_i(\xi))A = \varepsilon A - \varepsilon A \subset 2^{-1}(W_0 - W_0) = W_0$, since the sets A, W_0 are convex and W_0 is balanced. As $(w_z - v)(M) \subset W_0$, we get $(u - w_z)(M) \subset (u - v)(M) + (v - w_z)(M) \subset W_0 - W_0 = 2W_0 \subset W$. We need to consider two cases:

(i) If $0 \notin u(M)$, then $u^{-1}(Z \backslash \{0\}) = M \supset \operatorname{supp} w_z$, and so $u_W := w_z$ satisfies all required properties.

(ii) If $0 \in u(M)$, since $(1-\varepsilon)\varphi_i < \psi_i \ \forall i \in I$ and M is compact, it follows that $\exists \delta \in (0,1)$, such that $(1-\varepsilon)\varphi_i < (1-\delta)\psi_i \ \forall i \in I$ on M. Since $\operatorname{supp} v \subset u^{-1}(Z \backslash \{0\})$, Lemma 1 shows that $\exists \omega \in C_{\mathbf{R}}(T) \otimes C_{\mathbf{R}}(S)$, with $0 \leqslant \omega < 1$, $\omega(\xi) > 1 - \delta \ \forall \xi \in \operatorname{supp} v, \operatorname{supp} \omega \subset u^{-1}(Z \backslash \{0\})$. Now define $u_W := \omega \cdot w_z \in (C_\Gamma(T) \otimes X) \otimes (C_\Gamma(S) \otimes Y)$. Thus, $u_W(M) \subset \omega(M) \cdot w_z(M) \subset [0,1) \cdot \operatorname{co}(u(M)) \subset \operatorname{co}(u(M))$, $\operatorname{supp} u_W \subset \operatorname{supp} \omega \subset u^{-1}(Z \backslash \{0\})$. It remains to prove that $(u - u_W)(M) \subset W$. Fix $\xi \in M$. There are two subcases:

(a) If $\xi \in \operatorname{supp} v$, then we clearly have on M the pointwise inequalities $(1-\varepsilon)\varphi_i(\xi) < (1-\delta)\psi_i(\xi) < \omega(\xi)((1-\varepsilon)\varphi_i(\xi) + \varepsilon/|I|) \quad \forall i \in I$, and so $\omega(\xi) = \omega(\xi)\sum_{i \in I}((1-\varepsilon)\varphi_i(\xi) + \varepsilon/|I|) > (1-\varepsilon)\sum_{i \in I}\varphi_i(\xi) = 1 - \varepsilon$. Hence, $(u_W - v)(\xi) = (\omega w_z - v)(\xi) = \sum_{i \in I}[\omega(\xi)\psi_i(\xi) - (1-\varepsilon)\varphi_i(\xi)]z_i - \varepsilon \sum_{i \in I}\varphi_i(\xi)z_i + \omega(\xi)(1 - \sum_{i \in I}\psi_i(\xi))z \in \sum_{i \in I}[\omega(\xi)\psi_i(\xi) - (1-\varepsilon)\varphi_i(\xi)]A - \varepsilon A + \omega(\xi)(1 - \sum_{i \in I}\psi_i(\xi))A = [\omega(\xi) - (1-\varepsilon)]A - \varepsilon A \subset \varepsilon A - \varepsilon A \subset 2^{-1}(W_0 - W_0) = W_0$, and so $(u - u_W)(\xi) = (u - v)(\xi) + (v - u_W)(\xi) \in W_0 - W_0 = 2W_0 \subset W$. We conclude that $(u - u_W)(\operatorname{supp} v) \subset W$.

(b) If $\xi \in M \backslash \operatorname{supp} v$, then it is clear that $u(\xi) = (u - v)(\xi) \in W_0$, $u_W(\xi) = \omega(\xi)(w_z - v)(\xi) \in \omega(\xi)W_0 \subset W_0$. Thus, $(u - u_W)(\xi) \in W_0 - W_0 = 2W_0 \subset W$. Hence, $(u - u_W)(M \backslash \operatorname{supp} v) \subset W$.

From (a) and (b), it follows that $(u - u_W)(M) \subset W$. Therefore, u_W satisfies all required properties. □

In the particular case $X = Y = \Gamma$ we get

Corollary 1. *For every function* $u \in C_\Gamma(T \times S)$, *there is a sequence* $(u_n)_{n \geqslant 1} \subset C_\Gamma(T) \otimes C_\Gamma(S)$, such that $u_n \xrightarrow{\text{u.}} u$ and

$$u_n(T \times S) \subset \operatorname{co}(u(T \times S)), \quad \operatorname{supp} u_n \subset u^{-1}(\Gamma \backslash \{0\}) \ \forall n \in \mathbf{N}^*.$$

2.2. The case of $E_0 \subset C_0(T, X) \otimes C_0(S, Y) \subset C_\infty(M, Z)$

Theorem 3. *If* $u \in C_\infty(M, Z)$, *then for all* $W \in \mathscr{V}_Z(0)$ *and compact* $K \subset M$, *there exists an approximant* $u_{W,K} \in E_0$, *such that*

$$(u - u_{W,K})(M) \subset W, \quad u_{W,K}(M) \subset \operatorname{co}(u(M) \cup \{0\}),$$

$$u_{W,K}(K) \subset \operatorname{co}(u(M)), \quad \operatorname{supp} u_{W,K} \subset u^{-1}(Z \backslash \{0\}).$$

Proof. We can assume that $u \not\equiv 0$, that is $\exists \xi_0 \in M$, with $u(\xi_0) \neq 0$. Fix $W \in \mathscr{V}_Z(0), K$ compact in M and set $F := K$ if $0 \notin u(K)$, and $F := \{\xi_0\}$ if $0 \in u(K)$. Since $0 \notin u(F)$ and $u(F)$ compact, $\exists W_0 \in \mathscr{V}_Z(0)$, such that $W_0 \subset W$, W_0 open and convex and $u(F) \cap W_0 = \emptyset$, that is $F \subset u^{-1}(Z \backslash W_0)$. For every $A \subset M$, set $A_T := \pi_1(A)$, $A_S := \pi_2(A)$, $A_\pi := A_T \times A_S$. For $H := u^{-1}(Z \backslash W_0)$, $D := u^{-1}(Z \backslash 2^{-1}\overline{W_0})$, $L := u^{-1}(Z \backslash 2^{-1} W_0)$, we have $F \subset H_\pi \subset D_\pi \subset L_\pi \subset M$, H_π and L_π are compact, D_π is open. Since $L_\pi = L_T \times L_S$ and $u_{|_{L_\pi}} \in C_Z(L_\pi)$, by Theorem 2, $\exists v = \sum_{i \in I} a_i \otimes b_i \in C_X(L_T) \otimes C_Y(L_S)$, such that $(u - v)(L_\pi) \subset W_0, v(L_\pi) \subset \mathrm{co}\,(u(L_\pi))$ and $\operatorname{supp} v \subset L_\pi \cap u^{-1}(Z \backslash \{0\})$. Since H_T, L_T are compact, D_T is open and $H_T \subset D_T \subset L_T \subset T$, by Urysohn's lemma, $\exists \varphi : T \to [0, 1]$ continuous, with $\varphi_{|_{H_T}} \equiv 1, \operatorname{supp} \varphi \subset D_T$. Similarly, $\exists \psi : S \to [0, 1]$ continuous, such that $\psi_{|_{H_S}} \equiv 1, \operatorname{supp} \psi \subset D_S$. Define $\omega := \varphi \otimes \psi : M \to [0, 1]$. Hence, $\omega_{|_{H_\pi}} \equiv 1$ and $\operatorname{supp} \omega \subset D_\pi$. Finally, define the function

$$w : M \to Z,\ w(\xi) = \begin{cases} (\omega v)(\xi) & \text{if } \xi \in L_\pi, \\ 0 & \text{if } \xi \in M \backslash L_\pi. \end{cases}$$

Obviously, $\operatorname{supp} w \subset \operatorname{supp} v \subset u^{-1}(Z \backslash \{0\})$, $w_{|_{H_\pi}} = v_{|_{H_\pi}}$, $w(F) = v(F) \subset \mathrm{co}(u(M))$, $w(M) \subset \omega(L_\pi) \cdot v(L_\pi) \cup \{0\} \subset [0, 1] \cdot \mathrm{co}(u(M)) \subset \mathrm{co}(u(M) \cup \{0\})$ and $w \in E_0$. We next show that $(u - w)(M) \subset W_0$. We clearly have

$$(u - w)(M) = (u - w)(H_\pi) \cup (u - w)(L_\pi \backslash H_\pi) \cup (u - w)(M \backslash L_\pi).$$

But $(u - w)(H_\pi) = (u - v)(H_\pi) \subset W_0$ and $(u - w)(M \backslash L_\pi) = u(M \backslash L_\pi) \subset u(M \backslash L) \subset 2^{-1} W_0 \subset W_0$. For $\xi \in L_\pi \backslash H_\pi, (u - w)(\xi) = (1 - \omega(\xi))u(\xi) + \omega(\xi)(u - v)(\xi) \in (1 - \omega(\xi)) W_0 + \omega(\xi) W_0 = W_0$. Hence, $(u - w)(M) \subset W_0 \subset W$. If $F \neq K$, then $0 \in u(K)$, and so $w(K) \subset w(M) \subset \mathrm{co}(u(M) \cup \{0\}) = \mathrm{co}(u(M))$. We conclude that $u_{W,K} := w$ satisfies all required properties.

In the particular case $X = Y = \Gamma$ we get

Corollary 2. *If $u \in C_\infty(T \times S, \Gamma)$, then for every compact $K \subset M$, there is a sequence $(u_n)_{n \geqslant 1} \subset C_0(T, \Gamma) \otimes C_0(S, \Gamma)$, such that $u_n \xrightarrow{\text{u.}} u$ and*

$$u_n(T \times S) \subset \mathrm{co}(u(T \times S) \cup \{0\}), \quad u_n(K) \subset \mathrm{co}(u(T \times S)),$$

$$\operatorname{supp} u_n \subset u^{-1}(\Gamma \backslash \{0\}) \forall n \in \mathbf{N}^*.$$

2.3. An application: the density of $C_0(T, X) \otimes C_0(S, Y)$ in $C_0(M, Z)$ with respect to the inductive limit topology

Theorem 4. *If $u \in C_0(M, Z)$, then for every $V \in \mathscr{V}_{C_0(M,Z)}(0)$ with respect to the inductive limit topology, there exists $u_V \in E_0$, such that*

$$u - u_V \in V, \quad u_V(M) \subset \mathrm{co}(u(M)),\ \operatorname{supp} u_V \subset u^{-1}(Z \backslash \{0\}).$$

Proof. We can assume that $0 \in u(M)$, since otherwise M is compact and the conclusion is given by Theorem 2. Fix $V \in \mathscr{V}_{C_0(M,Z)}(0)$ and set $K :=$

$\operatorname{supp} u$, $C_0(M,Z)_K := \{v \in C_0(M,Z) \mid \operatorname{supp} v \subset K\}$. Since $V \cap C_0(M,Z)_K$ is a neighborhood of the origin in $C_0(M,Z)_K$ with respect to the uniform con vergence topology, it follows that $\exists W \in \mathscr{V}_Z(0)$, with $\{v \in C_0(M, Z)_K \mid v(M) \subset W\} \subset V \cap C_0(M,Z)_K$. Now Theorem 3 shows that $\exists v \in E_0$, such that $(u-v)(M) \subset W$, $v(M) \subset \operatorname{co}(u(M) \cup \{0\}) = \operatorname{co}(u(M))$, $\operatorname{supp} v \subset u^{-1}(Z \setminus \{0\}) \subset K$. We thus get $u - v \in C_0(M,Z)_K$, $(u-v)(M) \subset W$, and so $u - v \in V$. Hence, $u_V := v$ satisfies all required properties. □

Corollary 3. *E_0, and consequently $C_0(T,X) \otimes C_0(S,Y)$, is dense in $C_0(M,Z)$ with respect to the inductive limit topology. Moreover, if X and Y are metrizable, then this density is sequential.*

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